Participatory Budgeting with Multiple Resources

Nima Motamed[1], *Arie Soeteman*[2], Simon Rey[2], Ulle Endriss[2]

September 15, 2022 | EUMAS 2022 | Düsseldorf, Germany

[1] Intelligent Systems, Utrecht University

[2] Institute for Logic, Language and Computation, University of Amsterdam

What's so good about PB?

Dit plan wordt uitgevoerd

Vergroenen openbare ruimte

Geuzenveld, Slotermeer, Sloterdijken

> Lees meer

€ 65.000 | 1462 stemmen

Dit plan wordt uitgevoerd

Opknappen Natuurspeeltuin Nature...

Geuzenveld, Slotermeer, Sloterdijken

> Lees meer

€ 50.000 | 1216 stemmen

Dit plan wordt uitgevoerd

Bloementuin in het Sloterpark

Geuzenveld, Slotermeer, Sloterdijken

> Lees meer

€ 5.000 | 1207 stemmen

Dit plan wordt uitgevoerd

Bijeenkomsten voor eenzame ouderen

Geuzenveld, Slotermeer, Sloterdijken

> Lees meer

€ 18.780 | 1000 stemmen

Dit plan wordt uitgevoerd

Bewoners Restaurant Armoedebestr...

Geuzenveld, Slotermeer, Sloterdijken

> Lees meer

€ 10.000 | 981 stemmen

Dit plan wordt uitgevoerd

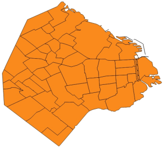
Voedselbos in het Sloterpark

Geuzenveld, Slotermeer, Sloterdijken

> Lees meer

€ 20.000 | 948 stemmen

Introducing Multiple Resources



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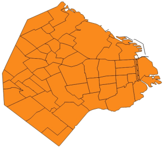

Officials often need to interfere in the process (Goldfrank, 2007)

MRPB has been recognized as an important challenge (Haris Aziz & Nisarg Shah, 2020)

Usual PB framework

The 'usual' PB framework often looks like this:

- Set P of projects
- Cost function $c : P \to \mathbb{N}$
- Budget limit $b \in \mathbb{Z}_+$
- Each voter i submits some sort of ballot A_i, making a profile $\boldsymbol{A} = (A_1, \ldots, A_{|N|})$

Project set $S \subseteq P$ is *Feasible* if $\sum_{p \in S} c(p) \leq b$

Our framework

A *d-resource PB scenario* is a tuple $\langle P, H, @ \rangle$:

- P is a set of projects
- H is a vector of cost functions $c_c : P \to \mathbb{N} \cup \{0\}$ for $k = 1 \dots d$
- $@$ is a vector of budget limits $b_c \in \mathbb{N}$ for $k = 1 \dots d$

A set $S \subseteq P$ is *feasible* if $\sum_{q \in \sim} c_c(p) \leq b_c$ for all $k = 1 \dots d$.

Voters $i \in \{1, \dots, N\}$ submit approval ballots $A_{\_} \subseteq P$

Approval ballots make up a *profile* $8 = (A_1, \dots, A_j)$

Other constraints & relations to other frameworks

Distributional: spend at most $2\ [0;1]$ of b_c on $X \quad P$

Incompatibility: not all projects in $X \quad P$ can be realised simultaneously

Dependency: p can only be realised if all projects in X are realised

Other constraints & relations to other frameworks

Distributional: spend at most 2 $[0;1]$ of b_c on X P

Add k with $b_c = b$ $b_c c$, and $c_c(p) = {}_{q2\pm}$ $c_c(p)$

Incompatibility: not all projects in X P can be realised simultaneously

Add k with $b_c = jXj$ 1 and $c_c(p) = {}_{q2\pm}$

Dependency: p can only be realised if all projects in X are realised

Add k with $b_c = 1$, $c_c(p) = jXj + 1$, and $c_c(q) = 1$ for all $q\ 2\ X$

Mechanisms

A mechanism is a function F that takes as input scenarios $hP; \mathtt{H}; @i$ and profiles 8 and returns feasible set $F(P; \mathtt{H}; @; 8) \quad P$

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- F_{greedy}: Go through projects in order of approval score, adding them to the outcome set one by one while skipping projects making outcome infeasible
- F_{max} returns feasible set with maximal approval score

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- F_{greedy}: Go through projects in order of approval score, adding them to the outcome set one by one while skipping projects making outcome infeasible
- F_{max} returns feasible set with maximal approval score
- F_{load} proceeds in steps: at each step, chooses the project minimizing the load (cost) carried by the worst-off voter

Axioms

Proportionality

All projects in set S are selected if for all k 2 R:

$$\frac{jf_{\_}2\ i\ ;8_{\_} = \sim gj}{j} \qquad \frac{H_c(\sim)}{@_c}$$

Weak axiom only guarantees this if $jSj = 1$

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(Approximate) Strategyproofness

For truthful ballot $S\_$, $F(8)\ 6\_\ F(A\ \_;S\_)$

Approximate: for some p 2 P: $F(8)\ 6\_\ F(A\ \_;S\_)\ [\ fpg$

Here we define different preferences _: prefer a Superset, or also an outcome that is better w.r.t. all resources (Paretian)

Proportionality

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Actually, our definitions are parameterized by a set *R* of relevant resources, giving more fine-grained analysis (and slightly different definitions)

Results

	Subset Preferences	Paretian Preferences	Paretian Preferences if $R = f1 ::: dg$
Greedy	✔	❌	✔
Max	❌	❌	❌
Load Balancing	❌	❌	❌

Approximate Strategyproofness

	Strong	Weak
Greedy	❌	❌
Max	❌	❌
Load Balancing	✔	✔

Proportionality

No mechanisms are strategyproof (even for $d = 1$)

Results

An impossibility result:

Theorem

Let $d > 1$, $m > b_c > 3$ *for some resource* k*, then no mechanism can guarantee both weak proportionality and strategyproofness against Paretian voters for* d*-resource PB scenarios with budgets* $(b_1; : : : ; b_c; : : : b_L)$ *and* m *projects.*

Basecase is generated using a SAT-solving approach

Computational analysis

F_{greedy} and F_{load} are polytime computable

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For $F_{\max}$ multiple decision problems:

Definition (MaxAppScore)

Instance: PB scenario $hP;$H$;@i$, profile 8, target K 2 N

Question: Is there feasible S P with approval score at least K?

(MaxAppScore$_L$ restricts to d-resource scenarios)

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(MaxAppScore$_L$ restricts to d-resource scenarios)

MaxAppScore$_1$ (and $F_{\max}$ in single-resource case) is polytime computable per Talmon & Faliszewski (2019);

MaxAppScore is strongly NP-hard;

MaxAppScore$_L$ for $d > 2$ is weakly NP-hard, and $F_{\max}$ is pseudo-polytime computable with restriction to d

Wrapping up

Summing up:

- Initiated the systematic study of PB with multiple resources
- New setting has significantly increased expressive power
- Mechanisms from single-resource setting largely carry over nice axiomatic & algorithmic properties

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What's next?

- Strengthen the results to e.g. other voter preferences, and other notions of proportionality
- Explore the introduction of negative costs
- Eventually implement multi-resource PB in real-world PB exercises

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Thank you!

The load-balancing mechanism

For set $R \quad \{1, \dots, d\}$ of relevant resources

Build outcome S in rounds. At each round, add a project that maintains feasibility of outcome S and minimises $\max_{c \in |} y_c$, where y_c is computed by linear program with variables $x_{\_,c,q}$

$$\min y_c \text{ where } y_c > \frac{1}{b_c} \sum_{q \in \sim} x_{\_,c,q} \text{ for all } i \in N \text{ with}$$

$$\sum_{\_ \in i} \quad _{q \in 8\_} \; x_{\_,c,q} = c_c(p) \text{ for all } p \in S, \text{ and } x_{\_,c,q} > 0$$

Intuitively, $x_{\_,c,q}$ is the part of the cost $c_c(p)$ 'shouldered' by voter i